CONTRIBUTION TO THE THEORY OF THERMAL INTERACTION BETWEEN THE POWDER COMBUSTION ZONE AND THE POWDER-METAL CONTACT

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A method is proposed in [1] for studying experimentally the conditions for powder extinction, in which thermal interaction between the combustion front and the metal-powder contact is used for creating extinction conditions in the combustion zone (method of "freezing" the combustion zone). Cylindrical powder samples with Plexiglas-coated lateral surfaces, placed on a massive copper plate, were burned in the experiment. The powder was ignited at the free end face of the sample. Since at the moment of ignition the distance between the combustion zone and the surface of the metal-powder contact is much greater than the characteristic thickness of the thermal layer in the powder, the cooling effect of the metal (high thermal conductivity) has almost no effect on the combustion process during the initial burning phase, so that the burning process becomes almost stationary shortly after ignition. As the combustion front approaches the metal-powder contact, the influence of the (high) thermal conductivity of the metal on the conditions in the combustion zone continues to increase. Heat removal from the combustion zone increases, the temperature gradient at the surface of the k-phase increases, the combustion conditions become nonstationary, the burning rate changes, and extinction occurs at a certain distance from the contact. On the copper plate there remains a layer of unburned powder, whose thickness depends on the initial temperature of the powder and on the gas pressure within the volume in which combustion occurs. In a series of experiments performed with powder samples with the same initial temperature, it was established that the pressure dependence of the thickness of the unburned powder layer can be described by the formula

$$\ln h = A - \nu \ln p \tag{1}$$

where h is the thickness of the powder residue, p is the pressure, A is an experimental constant, and ν is an experimental constant equal to the exponent in the power-law relation between the stationary burning rate and pressure.

The theory underlying the empirical relation (1) is proposed below. The experimental conditions are such that the propagation of the combustion front over the powder can be safely considered to be one-dimensional. An idealized picture of the mutual position of the combustion front and the metal-powder contact is shown schematically in the figure. The combustion front moves from the direction of positive values of x.

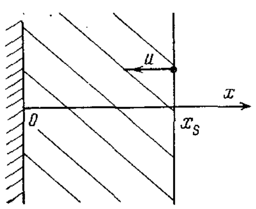

Fig. 1

The surface of the metal-powder contact coincides with the plane x=0 in such a way that the region x< 0 is occupied by the metal, and the region $0 < x < x_s$ by the powder. The heat conductivity of the metal is assumed to be large (infinite in the limiting case) compared to that of the powder. In the limiting case, the temperature of the contact surface may be assumed to have a constant value equal to that of the initial temperature T_0. (To justify this assumption, the volume of the metal disk must be sufficiently large; otherwise, the total heating of the disk should be taken into account.)

We formulate the problem of unsteady burning of a flat layer of powder. The variation of the powder temperature T(x, t) is described by the

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equation

$$\frac{\partial T}{\partial t}=\varkappa\frac{\partial^2 T}{\partial x^2}\quad (0\leqslant x\leqslant x_s(t)) \tag{2}$$

where x, t are a coordinate and time, respectively; $\varkappa$ is the thermal conductivity coefficient; and x_s (t) is the coordinate of the powder surface which varies due to the propagation of the combustion front. Initial and boundary conditions must be obtained for Eq. (2). In virtue of the adopted hypothesis about a high thermal conductivity and high integral specific heat of the metal disk, the condition of constant temperature

$$x=0,\ T=T_0 \tag{3}$$

must be fulfilled at any moment of time at the metal-powder contact.

The boundary condition at the burning surface $x=x_s(t)$ depends on the type of combustion model adopted.

We assume that powder combustion is described by Ya. B. Zel'dovich's theory [2], and also that the temperature at the burning powder surface remains constant during the entire combustion process

$$x=x_s(t),\ T=T_s. \tag{4}$$

The velocity of motion of the burning surface is equal to the burning rate u(t);

$$\frac{dx_s}{dt}=-u. \tag{5}$$

As in [2], we assume that the burning rate under nonstationary conditions depends on the pressure and temperature gradient at the burning surface inside the powder and that this dependence is the same for nonstationary and stationary conditions.

With this assumption, the derivation of an explicit expression for the nonstationary burning rate $u(\varphi, p)$ must be based on a stationary dependence of the burning rate on the pressure and initial temperature (e.g., an empirical dependence) and also on a nonstationary relation between the burning rate and the temperature gradient at the burning surface. An empirical dependence of the burning rate of powder on pressure and initial temperature usually can be represented in the form

$$u_0(p,\ T_0)=f(T_0)u_1p^{\nu}, \tag{6}$$

where u_0 is the burning rate of powder under stationary conditions, T_0 is the initial temperature, p is the pressure, ν and u_1 are experimental constants, and $f(T_0)$ is a known function which may be given, for example, in graphical form.

The temperature gradient at the burning surface under stationary burning conditions is related to the burning rate by the formula

$$\varphi\equiv\left(\frac{\partial T}{\partial x}\right)_s=\frac{u_0}{\varkappa}(T_s-T_0). \tag{7}$$

After eliminating T_0 in (6) and (7), we get a dependence of the burning rate on the temperature gradient at the burning surface and on pressure, which holds also for nonstationary conditions

$$u(\varphi,\ p)=f\left(T_s-\frac{\varkappa\varphi}{u_0}\right)u_1p^{\nu}. \tag{8}$$

From Eqs. (5) and (8) it follows that under nonstationary conditions at constant pressure, the time dependence of the displacement rate of the moving boundary, x_s, is defined by the dependence of the temperature gradient φ at the burning surface of the powder.

The temperature gradient at the burning surface varies during the combustion process but cannot exceed a certain critical value φ^*, which represents the maximum value of the gradient observed under stationary burning conditions. By using the condition of maximum, i.e., by differentiating (7) over T_0 with allowance for Eq. (6), and equating the results to zero, we arrive at the equation

$$\frac{d\ln f(T_0)}{dT_0}=\frac{1}{T_s-T_0}. \qquad (9)$$

The solution to Eq. (9), which is $T_0=T_0^*$, defines the minimum initial temperature at which a steady burning regime can still exist. Since the temperature gradient at the burning surface reaches its maximum value in this regime, the critical gradient φ^* can be obtained from Eqs. (6), (7) by substitution of $T_0 = T_0^*$:

$$\varphi^*=\frac{u_1p^\nu}{\varkappa}f(T_0^*)(T_s-T_0^*). \qquad (10)$$

Combustion ceases at the moment where the temperature gradient at the burning surface reaches the critical value φ^*. Consequently, the condition for extinction has the form

$$\left(\frac{\partial T}{\partial x}\right)_s=\varphi^*=\frac{u_1p^\nu}{\varkappa}f(T_0)(T_s-T_0^*). \qquad (11)$$

In order to finally formulate the extinction problem, it is necessary that an initial condition, which describes the temperature distribution in the powder at the initial moment of time, be introduced to Eq. (2), the boundary conditions (3), (4), the Eqs. (5), (8) (which define the law that governs the motion of the moving boundary), and to the condition for extinction.

For ideal formulation of the problem, it may be assumed that the combustion front propagates from infinity. Such a propagation is accompanied by a heat wave which moves in front of the combustion front, and temperature distribution of which is described by the so-called Michelson temperature profile

$$T(x)=T_0+(T_s-T_0)\exp\frac{u_0(x-x_s)}{\varkappa_i}. \qquad (12)$$

It is obvious that an infinite powder layer ($0 < x < -\infty$) burning at a finite rate will burn indefinitely, in which case the problem loses its sense. We shall therefore replace the ideal formulation of the problem by an approximate one. We assume that at a moment of time $t=0$, the powder layer had a finite thickness $l=x_s(0)$, that the temperature distribution in the powder was of the form (12), and that the surface temperature of the metal-powder contact was not T_0 but rather T_0^+, described by (12), i.e.,

$$x=0,\quad T(0)=T_0+(T_s-T_0)\exp\frac{-lu_0}{\varkappa} \qquad (13)$$

$$t=0,\quad x_s(0)=l,\quad T(x,0)=T_0+(T_s-T_0)\exp\frac{u_0(x-l)}{\varkappa}. \qquad (14)$$

From formula (14), it may be seen that by increasing l, the difference between the temperatures T_0 and T_0^+ can be reduced, thereby increasing the accuracy of the adopted assumption concerning the value of the temperature at the surface of the metal-powder contact.

With the assumptions introduced, the problem of a plane powder layer burning on a metallic substrate reduces to the solution of Eq. (2) with the boundary conditions (4) and (14), and the initial condition (13) for a given law of motion of the moving boundary, described by Eqs. (5), (8). A solution to this problem should permit determination of the temperature profile in the powder, the position of the burning surface, and the burning rate at any moment of time down to the moment of extinction, where the temperature gradient at the burning surface becomes critical. This will also yield the thickness h of the unburned powder layer.

The problem formulated is a complex nonlinear one which does not lend itself to analytical solution. A relation between the thickness of the powder residue and pressure can be derived without obtaining an analytical solution to the problem if use is made of similarity and dimensionality techniques. We write the problem (2), (4), (14), (13), (5), (8), (11) in dimensionless form

$$\frac{\partial\vartheta}{\partial\tau}=\frac{\partial^2\vartheta}{\partial\xi^2} \quad (0<\xi<\xi_s(\tau)) \tag{15}$$

$$\vartheta(\xi_s,\tau)=1,\ \vartheta(0,\tau)=e^{-L},\ \vartheta(\xi,0)=e^{\xi-L},\ \xi_s(0)=L \tag{16}$$

$$\frac{d\xi_s}{d\tau}=-w, \quad w=F\left(w,\left(\frac{\partial\vartheta}{\partial\xi}\right)_s, \quad a_1\right), \quad \left(\frac{\partial\vartheta}{\partial\xi}\right)_s=k \tag{17}$$

with the aid of the following dimensionless combinations:

$$\vartheta=\frac{T-T_0}{T_s-T_0}, \quad \tau=\frac{tu_0^2}{\varkappa}, \quad \xi=\frac{xu_0}{\varkappa}, \quad \xi_s=\frac{x_s u_0}{\varkappa}$$

$$L=\frac{lu_0}{\varkappa}, \quad \delta=\frac{hu_0}{\varkappa}, \quad w=\frac{u}{u_0}, \quad a_1=\frac{T_s}{T_0}$$

$$F\left(w,\left(\frac{\partial\vartheta}{\partial\xi}\right)_s, \quad a_1\right)=\frac{f\left[T_0\left(a_1-\left(a_1-1\right)\frac{1}{w}\left(\frac{\partial\vartheta}{\partial\xi}\right)_s\right)\right]}{f(T_0)}$$

$$k=\frac{f(T_0^*)(T_s-T_0^*)}{f(T_0)(T_s-T_0)} .$$

It should be noted that the dimensionless combinations do not contain the pressure.

It is evident that in the general case the dimensionless temperature distribution obtained from (15)-(17) will depend on the dimensionless combinations

$$\vartheta=\Phi(\xi,\tau,l,k,a_1) . \tag{18}$$

At the same time the dimensionless thickness δ of the unburned powder residue, of interest to us, should be independent of the dimensionless variables ξ, τ, so that

$$\delta=\delta(L,k,a_1) \tag{19}$$

Moreover, from the conditions of the problem, it follows that the dependence of ϑ and δ on L may be neglected, since for a properly performed experiment (sufficiently large initial thickness of the powder layer), L has no influence on the experimental results. Hence, one may write

$$\delta=\delta(k,a_1) . \tag{20}$$

The dimensionless parameters k, a_1 depend on the initial temperature T_0 and are independent of pressure p. This means that in experiments aimed at studying extinction, the parameters k, a_1 remain constant when the initial temperature is kept constant (only the pressure is varied). This is why in a series of tests in which the pressure is varied while the temperature is kept at the initial level, the dimensionless thickness δ of the unburned powder layer, as defined by Eq. (20), will remain constant;

$$\delta=\text{const}\equiv C . \tag{21}$$

From (21), it is easy to establish a relation between the thickness h of the powder layer and the pressure. Making use of the definition of the dimensionless parameter δ, together with Eq. (6), from (21) we get

$$h=\frac{\varkappa C}{f(T_0)u_1 p^{\nu}}, \quad \ln h=A-\nu\ln p \quad \left(A=\ln\frac{\varkappa C}{f(T_0)u_1}\right) . \tag{22}$$

It can be seen that the relation (22) obtained correlates well with the empirical relation (1). It can be shown that consideration of the substrate's finite thermal conductivity does not change Eq. (22).

It is noteworthy that the analysis performed makes it possible to obtain solely a relation between the thickness of the powder residue and pressure. A relation between the thickness of the powder residue and the initial temperature can be obtained apparently only from the actual solution of the problem formulated.

LITERATURE CITED

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